BUSINESS CYCLES AND ANIMAL SPIRITS
IN A CASH-IN-ADVANCE ECONOMY:
THE ROLE OF THE INTERTEMPORAL ELASTICITY
OF SUBSTITUTION REVISITED

This note examines the dynamical properties of a one-sector cash-in-advance constraint model with constant returns to scale. It is shown that, in opposition to available results, indeterminacy occurs for values of the intertemporal elasticity of substitution in consumption consistent with the bulk of empirical estimates. Furthermore, we find that sunspot shocks do not necessarily generate countercyclical movements in consumption. Considering simultaneously beliefs and technological disturbances, it turns out that the model performs as well as real sunspot models with increasing returns to scale in matching the business cycle.

Key words: Money, Indeterminacy, Sunspots, Business Cycle.

1. Introduction

Following the contributions of Benhabib and Farmer (1994) and Farmer and Guo (1994), a research field in macroeconomics has focused on models in which business cycles are driven by self-fulfilling changes in agents’ beliefs. Most studies assume that the households’ utility function is logarithmic in consumption, which is equivalent to setting the intertemporal elasticity of substitution (IES) in consumption equal to one. Noticeable exceptions are Bennett and Farmer (2000), Lloyd-Braga, Nourry and Venditty (2006) and Harrison (2001) who consider either non-separable preferences or a more generalized CRRA utility function. These works notably show a negative relation between the degree of increasing return needed for indeterminacy and the IES. Consequently, setting the IES significantly greater than one, they are able to generate indeterminacy with empirically plausible scale economies. Yet, this requisite is at odds with the empirical evidence which suggests that the IES is much lower than unity, many estimates being indeed below 0.5 (see, e. g. Kocherlakota (1996) and Campbell (1999)). The second
weakness of these models is their inability to match various moments of key macroeconomics variables. In particular, for reasonable values of the externality parameters, they generate a time series for consumption that is countercyclical, which is not consistent with the data.

The previous considerations cast some doubts on the empirical relevance of indeterminacy and expectations-driven business cycle. In this paper, we focus on monetary imperfections captured by a cash-in-advance (CIA) constraint on consumption in an economy with endogenous labor supply. More precisely, we study the basic (no externalities) monetary Real Business Cycle model of Cooley and Hansen (1989) with constant returns to scale extended to account for non-logarithmic utility in consumption. We establish that this constant returns to scale model exhibits indeterminacy for values of the IES in accordance with the bulk of empirical estimates, that is below 0.5. Numerical simulations indicate that fluctuations solely driven by sunspot disturbances are not necessarily accompanied by countercyclical movements in consumption.1 Allowing belief and productivity shocks, we show that this „simple“ one-sector model with constant returns to scale perform as well as more „complex“ real (one or two-sector) models with increasing returns to scale.

2. The Model
2.1. Environment
The economy consists of households, firms and a monetary authority. The representative household chooses sequences of consumption $c_t$, hours worked $l_t$, capital stock $k_{t+1}$ and cash balances $m_{t+1}$ to solve:

$$\max_{\{c_t, l_t, k_{t+1}, m_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma} - A^f l_t^i}{1 - \sigma} \right]$$

s.t. $c_t + k_{t+1} + \frac{m_{t+1}}{p_t} = w_t l_t + (r_t + 1 - \delta) k_t + \frac{m_t}{p_t}$

(1)

$$c_t \leq \frac{m_t}{p_t}$$

(2)

for $E$ the rational expectation operator, $A > 0$, $\sigma > 0$, $\chi > 0$, $\beta$ the discount factor, $\delta$ the depreciation rate of capital, $p_t$ the price level, $r_t$ the real return on capital and $w_t$ the real wage.

(1) is the usual intertemporal budget constraint; (2) is the cash-in-advance constraint (hereafter CIA). On the production side, the technology of the representative firm is described by the Cobb-Douglas production function:

$$z K^\alpha L^{1-\alpha}$$

for $L$ and $K$ the aggregate labor and capital factors, respectively; $\$z\$ is the state of technology which evolves as:

$$\log z_t = \rho_z \log z_{t-1} + (1 - \rho_z) \log z^* + \sigma_z \xi_t$$

(3)

where $\rho_z < 1$, $\sigma_z > 0$ and $\xi_t$ is a zero-mean i. i. d. random variable with unit variance. Markets being perfectly competitive, profit maximization implies that factors are paid according to their marginal productivities.

Lastly, as we do not study the effects of the monetary policy shocks, we assume

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1 Barinci and Chéron (2001) build on a related idea and demonstrate that a model with heterogeneous households and borrowing constraint outperform standard sunspots models in explaining business cycle facts, notably procyclical consumption.
that the monetary authority plays a fairly limited role: it supplies a constant quantity of money $M_t = M$.

2.2. Equilibrium

Let the CIA constraint holds with equality and consider the market clearing conditions. It is straightforward to see that an equilibrium is a sequence which satisfies:

\[ \frac{I^{x+a}}{z_i k_{b}} = \beta E_t \left[ \left( z_{b+1} \alpha k_{b+1}^{\alpha} I^{x+a}_{t+1} + 1 - \delta \right) \frac{I^{x+a}}{z_r k_{b+1}^a} \right] \]  
(4)

\[ \frac{A \alpha I^{x-a}}{z_i (1 - \alpha) k_{b}} = \beta E_t \left[ \frac{c_{t+1}}{c_i} \right] \]  
(5)

\[ k_{t+1} = z_i k_{b}^{\alpha} I^{x-a}_{t+1} + (1 - \delta) k_t - c_i \]  
(6)

It should be emphasized that if one assumes that the household’s utility is logarithmic in consumption the dimension of the equilibrium system (4)–(6) would actually be lowered. In fact, in such circumstances (5) would boil down to a static relation defining and the equilibrium is bound to be determinate.

3. Local Dynamics

In this section we carry out the analysis of the local (deterministic) dynamics of the equilibrium system (4)–(6) around its stationary solution. According to the usual procedure we study the first order Taylor expansion of the equilibrium system evaluated at the steady state. Letting $J$ denotes the Jacobian matrix of the linearized system and $T$, $\Sigma$ and $D$ be the trace, the sum of the principal minors of order two and the determinant of the $J$, respectively, we obtain:

\[ Q(\psi) = -\psi^3 + T \psi^2 - \Sigma \psi + D \]

\[ T = 1 + \beta^{-1} + \frac{1}{1 - \sigma} + \frac{\nu}{\chi + \eta} \]

\[ \Sigma = \beta^{-1} + \frac{1}{1 - \sigma} + \frac{1}{\beta (1 - \sigma)} + \nu \frac{1 + \chi}{(\chi + \eta) (1 - \sigma)} \]

\[ D = \frac{1}{\beta (1 - \sigma)} \]

for $\rho \equiv \beta^{-1} - 1 + \delta$, $\eta \equiv \beta \rho (1 - \alpha) + \alpha$ and $\nu \equiv \frac{\rho - \delta}{\alpha} \beta \rho (1 - \alpha) > 0$.

Since one variable is predetermined and the others are free, indeterminacy occurs when $J$ has at least two roots located inside the unit circle.

Case 1: the IES is greater than 1, then the equilibrium is bound to be locally unique.

Case 2: the IES is lower than 1, it follows that indeterminacy requires that

\[ \sigma > 2 + \frac{\nu \chi}{2(1 + \beta^{-1})(\chi + \eta) + \nu} \equiv 2 + \Delta \]  
(7)

Then, indeterminacy typically emerges for values of the IES lesser than 0.5. In opposition to available results (see the discussion in the introductory section), one sees that the values of the IES that place the economy within the indeterminacy region are in accordance with the recent empirical estimates (see, e.g., Campbell (1999)). It is worthy
to note that such low values are nowadays fairly standard in the RBC literature (see, e.g., King and Rebelo (1999) who set $\sigma = 3$).

We conclude that indeterminacy appears more likely empirically plausible is the current model than in real models which require "high" IES and elasticity of the labor supply in order to generate indeterminacy with realistic increasing returns.

4. Business cycle properties

It is well-known that some time series properties of real sunspot models are not consistent with the business cycles data. For example, for plausible degrees of increasing returns, they generate time series for consumption that are countercyclical (see, e.g., Benhabib and Farmer (1996) and Schmitt-Grohé (2001)). In fact, in a walrasian model, the marginal rate of substitution between consumption and leisure equates the real wage. As a consequence, beliefs shocks that shift the labor-supply schedule along the (downward-sloping) labor-demand schedule, tend to force consumption and hours worked to move in opposite directions. In the current model, as long as the CIA constraint is binding, the marginal rate of substitution between consumption and leisure does not equate the real wage. Thus, a spontaneous increase in consumption (optimistic beliefs) does not necessarily translates into a fall in hours worked.

More particularly, even though it is not currently necessary, as an infinite value for the labor supply elasticity is usually assumed in the literature, we set $\chi = 0$. Thus, indeterminacy results when $\sigma > 2$. Following King and Rebelo (1999), we fix $\sigma = 3$ (IES = 1/3). As a benchmark, we examine how the model responds to sunspot shocks. Table 2 shows that our "endogenous business cycle" (EBC) CIA model produces a procyclical consumption. Nonetheless, it suffers from two stringent weaknesses: the investment is countercyclical and the volatilities of consumption and investment relatively to that of output are hugely overestimated. These counterfactual results come from the fact that even though a sunspot shock induces a simultaneous increase in consumption and hours worked, the rise of hours is so low that it generates a quite small increase in output. Consequently, a strong increase in consumption is sustained by a strong decrease in investment: investment is countercyclical, and relative volatilities are overestimated. This actually suggests that technological disturbances (supply shocks) must be added in order for the model to be consistent with the data. As usual, technological parameters are set to $\rho_z = 0.95$ and $\sigma_z = 0.007$. In addition, since we now consider two sources of uncertainty, the covariance matrix between technology and belief shocks has to be calibrated.

Let $\rho_{e/zeta}$ denotes the correlation between beliefs and technological shocks. Table 2 compares several possible moments when the correlation parameter takes three values we calibrate $\sigma_z/\sigma_e$ so that the model replicates the relative standard deviation of consumption to that of output, for $\sigma_e$ the standard deviation of the belief shock. It is seen in Table 2 that our CIA model generates realistic aggregate fluctuations provided that the correlation between beliefs and technological disturbances is positive which is equivalent of saying that sunspots are overreactions to news about fundamentals. For comparison purposes, we report the dynamical properties of the Benhabib and Farmer’s (1996) model generated with $\rho_{e/zeta} = 1$ are also reported. One then can see that our monetary model with constant returns to scale performs as well as a more "intricate" two-sector real model with increasing returns.

5. Concluding remarks

This paper has examined a cash-in-advance one-sector model in which indeterminacy occurs for constant returns to scale and values of the intertemporal elasticity of substitution in consumption consistent with the bulk of empirical estimates. Indeterminacy appears then more likely empirically plausible in this model than in real (one and two-sector) models. However, the model was not found to endogenously produce a procyclical consumption in a satisfactory way. This supports the wisdom that animal spirits
(demand shocks) cannot be invoked solely to explain the business cycle. Whenever sunspots and technological disturbances (supply shocks) are simultaneously allowed, the model performs equally as well as existing real sunspot models with increasing returns.

Bibliography

